THE SEARCH OF CONTINUOUS GRAVITATIONAL WAVES: A SHORT REVIEW OF THE EXPERIMENTAL RESULTS.

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Until now the majority of the gravitational wave experiments collected data looking for signals of short duration, such as might be emitted during the collapse of massive star. With the exception of the Tokyo/KEK group and the resonant room temperature antenna GEOGRAV, the search for continuous radiation was only occasional. In particular, for the interferometric gravitational wave detectors, the experimental results reported in the literature, are the by product of an exercise practiced in order to understand how to handle the huge amount of data taken by the interferometers. However the discovery of milliseconds pulsars triggered a still increasing interest in the detection problem of continuous signals, provided that there are reasonable arguments concerning the possibility to detect these sources.

Here we will try to summarize the experimental results obtained using the resonant antennas and the interferometers. In both cases we will try to focus the attention on the specific analysis technique that have been developed.

The resonant antennas.

Before presenting the experimental results obtained with the resonant antenna let us state some introductory remarks.

Suppose that the frequency of the signal is near the resonant frequency of the detector and that the dominant noise is the resonant one (brownian and back-action noise). In the limit of null electronic (wide band) noise, the Signal to Noise Ratio (SNR) results to be frequency independent (infinite detection bandwidth). In fact, in this case the SNR is the ratio of the square of the transducer output with its typical lorentzian shape divided by the thermal noise spectrum which has the same frequency dependence. Thus, it is useful to distinguish the "response bandwidth" which coincides with the mechanical bandwidth of the oscillator, to the SNR bandwidth (or detection bandwidth).

In a real system the limit of zero electronic noise does not hold, but it is not surprising to recognize that the detection bandwidth is much wider than the mechanical one. For example in the present experimental apparatus the typical values are $\nu_o = 10^3$ Hz, the mechanical quality factor $Q \sim 5 \cdot 10^6$ that corresponds to a $\Delta\nu_o \simeq 2 \cdot 10^{-4}$ Hz, but a detection bandwidth of the order of few hertz around each resonance.

- *The CRAB pulsar: the effort of the Tokyo group to detect the g.w. emission.*

- H.Hirakawa, K. Narihara, M.K. Fujimoto J. Phys Soc. Jpn. 41,(1976) 1093
- T. Suzuki: "Search for continous grvitational wave from pulsars with resonant detector" in Gravitational wave experiments E. Coccia, G. Pizzella, F.Ronga Editors, World Sci. ,Singapore (1995)
- K. Tsubono: " Detection of Continous waves" in The detection of gravitational waves G. D. Blair editor , Cambridge University Press , Cambridge (1991)

Since 1976, Hirakawa at the university of Tokyo started an experimental effort to detect the gravitational emission at $2\nu_r = 60$ Hz, the double of the rotation frequency of the Crab pulsar. This effort was pursued by developing antennas of different geometry systematically improving the detection sensitivity. The basic work is the elegant study of the interaction of the g.w. with the fundamental vibration modes of a solid body that H. Hyrakawa developed by approaching the problem on the base of the group theory in analogy to the molecular vibration theory and the interaction with the e.m. field.

We summarize the history of this research dividing it in three phases: the first and the second phase are characterized by the use of room temperatures antennas while the third, CRAB III, is a cryogenic experiment carried on at 4.2 K in the high energy physics laboratory KEK of Tsukuba. The antenna of the first generation was a square plate with cuts of each side. The quadrupolar symmetry of the fundamental plate mode insures an efficient coupling with an impinging gravitational wave. In the second phase the geometry of the antenna was changed and this new design was adopted later for the cryogenic antenna of KEK. The new detector consists of two masses connected with a torsional bar. This geometry has the advantage to have a nodal plane available for the suspension. In this configuration the overall quality factor of the torsional mode results to be maximized: at liquid helium temperature they measured $Q = 4.8 \cdot 10^7$. The transduction system was based on the use of an electrostatic transducer followed by a low noise FET amplifier $(T_n = 30 \ \ mK)$.

As we discussed above, one can detect a signal force on the antenna within the detection bandwidth. We recall here that the maximum shift of frequency in the case of Crab pulsar induced by the Doppler effects is 36 mHz/years. To get a detection bandwidth larger than this value, the coupling between antenna and transducer system should be so high that the back action noise of the amplifier became significant, particularly in the case of high Q antenna. Thus, the strategy adopted was to have a coupling factor β (the ratio between the e.m. energy at the transducer output to the detector mechanical energy) rather low ($\beta \sim 1 \cdot 10^{-6}$). Then, they apply the method of electrostatic tuning to track the frequency variation of the arriving gravitational wave. In practice, a control signal set the transducer bias voltage so that, the detector frequency was corrected every 30 s in order to coincide with the signal frequency. They claim that, according to this procedure an error of tracking phase of a g.w. may be proportional to $\ddot{\nu}$, while if the detuning is at the border of the signal this implies a phase error proportional to $\dot{\nu}$. The detector output was sent to a Phase Sensitive Detector and the two quadrature outputs were stored in a computer memory. Some data were selected by eliminating those until the excitation after a pulse-like noise falls down to the level of thermal fluctuation. The PSD outputs were processed by computing for each possible polarization of the g.w. the h value. Data collected until May 1993 give a 1 σ upper limit of

$$h \leq 2 \cdot 10^{-22}$$

at 60 Hz for an averaged time of 1900 hours (Suzuki 1995).

- *The GEOGRAV gravitational-wave antenna.*

- S. Frasca, C. La Posta, Nuovo Cimento 14C, (1991) 235

The GEOGRAV gravitational-wave antenna was a room temperature aluminum cylindrical bar with a length of 3 m and a mass of 2280 kg and resonating at 858 Hz. This detector was located in Rome at the University La Sapienza and it was suspended horizontally with an angle of 61^o westward from south. The cylinder vibration was detected using a piezoelectric transducer mounted in a slot in the center section of the bar. The voltage signal was amplified via a low noise FET amplifier and then by a two-phase lock-in, driven by a frequency synthesizer at the resonance frequency of the antenna. The synthesizer, a Hewlett Packard 3325A with the option of high stability frequency reference, ensured an ambient stability of $5 \cdot 10^{-8}$ between 0 and 55 ^{o}C and an aging rate of $1 \cdot 10^{-7}$ per month, that, at the concerned frequencies , simulate a variation of the gravitational wave, source frequency of $|\dot{\nu}| \simeq 10^{-4}$ Hz/month. The antenna temperature was not stabilized, thus the resonance frequency was tracked using a non invasive method based on the thermal noise monitoring data. In practice the correction of frequency setting of the synthesizer was derived by counting during two hours the turns of the complex signal around the complex plane origin (the beats between the antenna signal and the lock-in reference signal).

In the period May 1984 and February 1988 the GEOGRAV antenna was on the air for 87 % of the total time. For the analysis of continuous signals, about the 95 % of all the available data in that period was considered, excluding only those data taken in coincidence with strong local disturbances or when the apparatus was malfunctioning. The data from the channels containing phase and amplitude of the detector output, were sampled at 1 Hz and were considered as a single complex time series. The data stream was divided in pieces of 8192 samples and the square modulus of the FFT of each piece (a periodograme) was computed, obtaining for each piece a bandwidth of 1 Hz around the resonance frequency and the resolution of 0.000122 Hz. So a spectral data-base was built, containing, for the entire period of almost four years, 12178 spectra . Because of the continuous change of the frequency the periodograms result to be asymmetrical. In order to look for (narrow) peaks in one periodogram, a good technique is to divide the periodogram by a smoothed version of it; this was accomplished by using a moving average on 100 samples of it. If the mean value 1 is subtracted, the fluctuations and the possible signals around the expected spectrum are obtained. In this way all the spectral data available cover, not continuously in time, a bandwidth of about 3.3 Hz between 856.0 and 859.3 Hz. The expected distribution of the values of a normalized spectrum is expected to be an exponential distribution with zero mean value and $\sigma = 1$. With collection of N spectra of this type on the same frequency range obviously we have a reduction of $\sigma = 1/\sqrt{N}$.

Two different procedures to estimate the received flux of gravitational radiation have been used. The first one consisted in searching for radiation from the whole sky, utilizing the Doppler effect, as well as the radiation pattern of the antenna, to identify the location of the source. The second one searched for radiation coming from a hypothesized spot of the sky (namely the center of the Galaxy and the Large Magellanic Cloud), correcting the frequencies of each normalized spectrum for the computed Doppler effect. In both cases the time modulation of the radiation pattern is used to check the plausibility of the location and to compute the other parameters of the source (linear polarization percentage and angle).

The Doppler effect plays an important role also in the case of the average of more normalized spectra. In order to analyze what is expected from a source located in a given spot of the sky, a computer simulation was performed obtaining a typical pattern with two peaks, that are due to the time when the Earth moved in the direction of the source or away from it: there the time derivative of the observed frequency was minimum. The distance between the two peaks gives the ecliptic latitude of the source, and the time of appearance of at least one of them gives its ecliptic longitude. In principle one can look for the presence of the two peaks in the data by proper matched filters, which, when applied to the mean normalized spectrum, enhance the signal-to-noise ratio. In the case of the real data of GEOGRAV, things result to be more involved, because they do not observe all the time the same frequency band, and so often we can find only one of the two peaks.

Means of 12178 normalized spectra, obtained by setting every spectrum at its frequency band have been computed. Under the hypothesis of no frequency variations in the observed source signal, the upper limit value in the range between 856.3 and 859.0 Hz becomes

$$h_l \simeq 3 \cdot 10^{-22}$$

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Moreover the matched filters on the monthly statistically normalized spectra have been applied and a list of peaks of the upper and lower types have been obtained, exceeding the threshold of 4 standard deviations. For each one of them, it has been looked for the presence of peaks shifted linearly in time of a small amount (± 0.15 Hz/y), occurring at a time distance which is multiple of six months, in cases the band was present at that time. Nothing meaningful was found. In the case of the largest peaks (May 1984, frequency 858.486 Hz, amplitude 5.7 σ and April 1987, frequency 858.415 Hz, amplitude 6.8 σ), there was no agreement with the expected time modulation pattern for a source located anywhere in the sky.

Finally a search from the center of the Galaxy and the Large Magellanic Cloud was completed.

For a given location of the source, it is possible to correct the Doppler effect on the frequencies of the normalized spectra, thus enhancing the SNR. This procedure was applied only in the cases of the Galactic Center and of the Large Magellanic Cloud.

In the case of the average of all the spectra, no peaks were found above the limit of

$$h_l \simeq 6 \cdot 10^{-22}$$

in the range of (856.3 - 858.8) Hz, both in the case of the Galactic Center and of the Large Magellanic Cloud. Also in this case data were segmented in pieces of one month to reduce the effect of the long term frequency variations. In the 46 spectra, any *unexplained* peak over 5 σ was found. Taking into account only the frequency ranges for which we have an average of at least 200 normalized spectra for all the months, it was derived a limit of

$$h_l \simeq 9 \cdot 10^{-22}$$

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- *Search of continuous signals with ALLEGRO data*

- E. Mauceli, P.Elcan, W.O. Hamilton, W.W. Johnson, M.P. McHugh, A. Morse, preprint of the Lousiana Gravitational Wave group available at Web site
http://phwave.phys.lsu.edu/www/papers/allegro$_{d}ata/cwgravity.html$

A more recent experimental result obtained with a resonant detector has been presented by the Louisiana group that operates ALLEGRO, an aluminum cylindrical antenna of 2.3 ton at 4 K equipped with a resonant transducer. There are two small bandwidth around which the antenna is sensitive, one for each normal mode of the system (896.80 Hz and 920.26 Hz). They applied the strategy to point the search towards two sky regions where one might reasonably expect to find a high density of sources: the globular cluster 47 Tucanae and the center of Galaxy. They performed the analysis under the simplest possible assumptions: stable emission frequency in its rest frame (negligible pulsar spinning). In this case only the knowledge of the Earth's motion and the antenna's radiation pattern are needed to determine the modulation of the frequency and the amplitude of the received signal. The search is performed in the two intervals of ±5 Hz around the two resonant modes of the detector with steps of 10 μHz. They assumed a signal shape which included the Doppler shift and amplitude modulation for the given epoch and direction, and they used it to create an optimum filter in the discrete frequency domain to be applied to the data. It is interesting to report here also the problems and solutions adopted to solve some difficulties that can be common to other similar experiments. We cite here two problem: the first was dealing with the handling of short term disturbances in the data stream. In this case some data were removed by interpolation and some by fitting and subtraction. When there were disturbances in the data longer than a few seconds the data segment was dropped out. Thus, the detector duty factor referred to the available data were reduced to about the 30 %. By applying the lock-in technique for data detection, the phase stability of the reference oscillator determines the limit of the maximum observation time and hence sets the minimum frequency bin size. The main experimental result obtained applying this data analysis procedure corresponds to a strain amplitude sensitivity of $4 \cdot 10^{-24}$. Moreover, in the case of the analysis carried on in the direction of the globular cluster 47 Tucanae , they discovered 4 peaks in the data at the frequencies

896.56142 Hz, 920.28043 HGz, 920.28070 Hz and 920.40582 Hz

with associated strain amplitudes

$$1.2 \cdot 10^{-23},\ 4.6 \cdot 10^{-24},\ 4.5 \cdot 10^{-24},\ 5.0 \cdot 10^{-24}.$$

The statistical meaning of this discovery was statistically evaluated at 4.6 σ for the lowest peak and 5.3 σ for the highest one. Such a large strain is inconsistent with simple consideration based on the energy conservation principle for sources with 10^7 or 10^8 year lifetimes: consistency can be recovered if it might be an emission for shorter periods of time.

- *The analysis of the Explorer data*

- P. Astone : "On the detection of monochromatic and stochastic gravitational waves with resonant gravitational wave detector" in the Proceedings of the Second Amaldi Conference on Gravitational wave experiments, CERN, Geneva, July 1996, Word Sci, Singapore in press.

Explorer is the resonant cryogenic antenna of 2300 kg installed at CERN, cooled at

2K using a liquid helium bath in super fluid regime. The first data taking of this detector was performed in 1983 and in practice since 1990 many long runs of data taking have been performed. The two resonances of the detector are at $\simeq$ 904 and $\simeq$ 921 Hz and the bandwidth is roughly of 1 Hz around these frequencies. The main attention on data analysis was related to the problem of burst detection , but recently a strategy for the detection of continuous signal on the detector bandwidths has been set up.

The strategy is based on the on-line construction of a frequency domain data base (FDDB) and on the software procedures operating on it, off-line and at different levels. The FDDB is composed taking the first half part of the FFT of 2N data. Each basic FFT in the data base is completely characterized allowing to take into account the data interruptions and the non-stationarity in the noise of the detector,being difficult real problems. The informations recorded in a header should be such to help during the analysis and can regard both environmental information (for example flags of the experimenter regarding the apparatus, i.e. normal operation, working around the system etc.) or the data quality. So it is possible to define different thresholds for vetoing the data or criteria to weight them differently in the analysis.

After a first step in the procedure which consists in the comparison of the spectrum bin content with a pre-defined threshold, a second order data base is created. It includes the candidate spectral lines. Then a study is performed on the candidates of successive spectra. At present this study consists in the fitting the successive line pattern with the expected frequency and amplitude modulation for gravitational waves emitted by sources in various locations in the sky.

At the time being the procedure has been applied just to 7 days of Explorer data, in order to check its efficiency. Combining 256 elementary FFT (without Doppler correction) just around the mode at 921 Hz, they get a noise spectral density

$$S_h{}^{1/2} \simeq 1 \cdot 10^{-21} \qquad /\sqrt{Hz}$$

in the bandwidth 921.2 - 921.4 Hz.

The interferometric gravitational wave detector

The data handling of the interferometric antennas is much more complex because of the huge amount of the housekeeping data. The data of an interferometric detector fully equipped requires a huge capacity storage. A detector will produce data at a rate of the order of 400 Mbytes/hours.

However, from the mathematical point of view, the main technique of analysis is the same of that used for the resonant antennas.

Here we present the main guidelines for the various analysis performed on experimental data and the eexperimental results.

- *Analysis of the data taken with 1.5 m M.I.T. interferometer*

- J. Livas :"Broadband search techniques for periodic sources of gravitational radiation", in Gravitational wave data analysis B.F. Schutz Ed., Kluwer Acad. Publisher Dordrecht,Nederland (1989)

The data taken with M.I.T. 1.5 m prototype gravitational wave antenna were also analysed in order to study the problems involved in the search of Pulsar signals. This work was presented in July 1987 at the NATO Advanced Resaerach Workhop on Data Analysis held just outside Cardiff. The main idea is to try to define strategies peculiar for this kind of broad band detector. Thus, this is one of the first work on this field. The raw data of the 1.5 m M.I.T.interferometer consists of the demodulated output of the interference fringe lock servo, digitized at either 20 kHz rate or 6 kHz. The digitization is triggered from a rubidium standard that is sychronized WWV. As mentioned above, limitations in the recording hardware restrict the length of a continuous data stream to approximately 15 minutes ($N = 2^{24}$ s $\simeq 16 \cdot 10^6$ points). The data collected in June 1985 consists of approximately 50 magnetic tapes collected over 6 days, representing roughly 10 % coverage of the total possible time. One half of the tapes were sampled at 20 kHz, the other half at 6 kHz. Eight of the 25 tapes sampled at 20 kHz were selected for the final analysis.

The first step in the analysis is to choose a value of the averaging bandwidth. For the statistical peak detection scheme to be valid, the statistics of the power in each resolution bin must agree with the theoretical Rayleigh distribution. The choice of an averaging bandwidth was made by constructing an experimental probability distribution for several different values of the averaging bandwidth and choosing the value which gave the best least squares fit to the theoretical distribution. The experimental distribution is computed by normalizing the value of the each frequency in the square root of the periodogram to the local standard deviation. The normalized points are then binned in a histogram which counts the number of points with a given normalized amplitude versus that normalized amplitude. A χ^2 fit of the experimental distribution to the theoretical distribution performed, and the value of the averaging bandwidth used in the final analysis was that which gave the lowest value of the χ^2 for the fit. For this data set, that value was 0.35 Hz.

Once the value of the averaging bandwidth has been chosen, the first step in analysis is straightforward: the spectrum of each piece of data is binned separately, and candidate signals are identified with the threshold criterion. A list of the possible signals in each piece is compiled, keeping track of the frequency of the signal, its magnitude in absolute units, and its value relative to the local standard deviation are recorded, along with an estimate of the number of adjacent frequency bins that also qualify as possible signals (a measure of the width of the peak). The complete spectrum of each piece extended from 0-10 kHz, but only the region from 2-5 kHz was searched. The low frequency cutoff was chosen at a point where the noise spectrum of the antenna begins to increase rapidly with decreasing frequency, and hence the sensitivity to gravitational radiation becomes small.

Moreover the signal expected at the detector from a stationary, purely sinousoidal g.w. source is modulated by the relative motion of the source and the antenna in two distinct ways: frequency modulation (FM), amplitude modulation (AM). Over the 6 days of data they expected to be dominated by the orbital motion of the Earth and the Doppler shift for the week may be characterized by a single slope for each direction of the sky which has a maximum physical value given by the ratio of the orbital velocity to the light velocity. A known value of the slope identifies a locus of points on the sky as possible source location.

Thus, the lists of possible peaks extracted from the data are then examined for a series of peaks whose center frequency changes during the week. They called the method for finding such a series a Doppler sieve. In practice, one piece of data is selected as the reference. If a peak in the list is actually an astrophysical signal, then it should appear in the lists of the other pieces, but with its center frequency shifted by a known amount. The sieve is applied by dividing the allowable range of Doppler slopes into discrete values. Each peak in the reference list is used to estimate the center frequency for each of those slopes that that peak would have if it were observed at the time the other data was collected. A weak signal is most likely to be seen only in the two most favorable observation windows. For each reference piece, the peaks that fall through the Doppler sieve are binned in histogram of the number of peaks with a given Doppler slope versus the slope. The peaks that manage to fall through the Doppler sieve are then associated with a region of the sky. The sky is divided into discrete bins, and for each bin in the region picked out by the Doppler slope, the expected pattern of signal strengths is determined. Map can be generated, each one for the various polarization. The actual strengths can be compared against the predicted pattern.

A total of three peaks out of $\simeq 2 \cdot 10^6$ frequency bins actually passed both the Doppler sieve test and the AM signal strength pattern. None of the peaks is a good candidate signal for a number reasons, but the primary reason is that all occured in a region of the spectrum in there was appreciable structure in the spectrum on the same scale size as the averaging bandwidth.

Full sky search procedure and single direction search procedure are similar and the final results of the two searches are the followiing: the full sky search did not see any periodic signals which produced a measured strain in the antenna above a level

$$h_{rms} = 4.6(\pm 0.9) \cdot 10^{-17} (\frac{2000}{\nu})^2$$

in a frequency band from 2-5 kHz. The indicated error is one standard deviation. The conversion of this measured strain into a source strength depends on the polarization and the direction of the source. For a source at the center of the galaxy, the limits translate into

$$h_{\oplus,rms} = 2.1(\pm 0.6) \cdot 10^{-16} (\frac{2000}{\nu})^2$$

$$h_{\otimes,rms} = 7.4(\pm 4.) \cdot 10^{-17} (\frac{2000}{\nu})^2$$

The corresponding limits for the single direction search are

$$h_{\oplus,rms} = 1.0(\pm 0.1) \cdot 10^{-16} (\frac{2000}{\nu})^2$$

$$h_{\otimes,rms} = 5.2(\pm .8) \cdot 10^{-17} (\frac{2000}{\nu})^2$$

Obviously, with more data, the increase in SNR could be much larger.

- *Pulsar search with the Garching interferometric detector*

- T. M. Niebaur, A Rudiger, R. Schilling, L. Schupp, W. Winkler K. Danzmann, Phys. Rev. D 47, (1993), 3106

In March 1989 a coincidence experiment was performed with the Garching and the Glasgow prototype laser interferometric gravity wave detector in order to verify their reliability over long time periods. 100 hours of data were taken between March 2-6, 1989. These data have been used also to search for a Pulsar within a restricted bandwidth of few Hz near 2 and 4 kHz in the direction of the 1987A Supernova. The original motivation for this analysis was provided by the reported optical sighting of a sub millisecond pulsar in the remnants of supernova 1987A by Kristian et al.. They observed an optical pulsar of frequency 1968.629Hz with significant power in higher harmonics. They also reported that the pulsar observations were improved by a transformation to mass center coordinates to remove effects of the Doppler shift. In the same paper, they speculated that dark matter could explain the fact that it was not observed 13 days later. Efforts made to duplicate the observation failed and it is now believed to have been caused by instrument error. Although the reported frequency and its second harmonic (2 and 4 kH) are well within the measurement bandwidth of the present gravitational-wave (GW) prototypes, any reasonable estimates of the expected strength from such a pulsar are still several orders of magnitude smaller than the sensitivity of these detectors. If, for a rough estimate, one assumes that the rotational energy of the reported pulsar (estimated by Kristian et al. as about $6 \cdot 10^{45}$ J) would be converted into GW radiation over only five years, the expected strain amplitude at the earth would be as low as $5 \cdot 10^{-25}$. This limit is still about three orders of magnitude too small to be seen using 100 h of data with the Garching prototype (assuming a constant noise background of about $10^{-19} \quad 1/\sqrt{Hz}$). Despite the poor chances of detecting gravitational radiation from pulsars in SN 1987A, they developed analysis techniques providing a practical framework in which the frequency domain statistics of the noise from the prototype detectors over long observation times could be studied and compared with the expected statistical distribution.

The first important by-products of the analysis was the learning of the existence of some periodic instrumental effects that have been removed later. The interferometer output spectrum result to be composed by several parts with different frequency behavior and, in order to improve the system dynamics, several trick were applied. The very high spectral contributions below 1 kHz, largely due to seismic noise, have been partly removed by an analog high-pass filter. The noise floor above 1 kHz becomes constant (white) and approaches the theoretical shot noise limit for the setup. The high-frequency portion of the spectrum was attenuated by two electronic low-pass filters providing sufficient anti-aliasing to ensure a good signal-to-noise ratio (SNR) up to 4 kHz. The main interferometer output was sampled at 10 kHz. The various housekeeping data channels raised the overall data rate to 20 kHz. Over 100 h about 10^{10} samples were written onto tape. Therefore it is important to keep the number of operations made on the full data set small to avoid excessive computational time. Even more problematic is the large amount of memory that is required to perform the efficient FFT routines. This limitation would rule out the possibility of making an FFT over the 100 h without the data reduction mentioned above.

The pulsar analysis performed by the Garching group is started by making a careful study of the time domain amplitude statistics and the possible correlations with the various housekeeping data which monitored servo conditions and seismic noise. Then, four distinct actions were recognized:

- application of a data compression algorithm ideally adapted to search a narrow bandwidth for periodic signals,
- Doppler shift corrections,
- polarization dependent demodulation.

In particular we describe the data compression and the method of Doppler shift corrections.

- Data compression

The authors claim that the most flexible data compression scheme is Complex Heterodyne Technique (CHT). The CHT requires a heterodyne (multiplication) of the original data with both a sine and a cosine function, with frequency ν_m chosen to lie at the center of the signal bandwidth. These two heterodyned series are then lowpass filtered (for reasons of antialiasing) before resampling.

It is usually more convenient to consider these two real quadratures as a single complex time series, the analytic signal, where the cosine and sine quadrature are taken to be the real and imaginary components, respectively. The analytic signal produced by the CHT is simply a frequency shifted version of the original data according to standard Fourier transform theory. Any signal $x(t) = x_o exp(-i\omega_o t)$ will be shifted to $x(t) = x_o exp[-i(\omega_0 - \omega_m)t]$. The data compression was made for two different frequencies near 2 and 4 kHz.

The last step of the CHT required low-pass filtering of each quadrature for each channel prior to the resampling. This was broken down into three successive stages of filtering and resampling. They have a first filter stage which was a simple moving boxcar average of 50 data points. These low-passed data were resampled such that neighboring averages overlapped by 50 %. The first step compressed each quadrature (for each channel) by a factor of 25, from 10 kHz to 400 Hz. The second stage compressed each quadrature by an other factor of 25 producing time series resampled at 16 Hz. The filter chosen for this step was a two-pole Butterworth infinite impulse response (IIR) digital filter with a cutoff frequency of about 2.53 Hz. The final stage compressed the data by another factor of 4 so that each quadrature was sampled at 4 Hz. This corresponds to a reduction in sampling rate by a factor of 2500. However, since they have to store two quadratures (sine and cosine) the data reduction factor is only 1250.

- Doppler shift correction

In general, the Doppler shift is removed by correcting the laboratory measurements by the time it would take a signal to travel from the detector to a rest frame as that of center of mass of planetary system. A monochromatic signal in the rest frame acquires in the laboratory frame a phase modulation due to the time difference δt between the detector and the barycenter:

$$\delta t = a_O cos(\Omega_O t + \Phi_O) + a_R cos(\Omega_R t + \Phi_R)$$

where Ω_O and Ω_R are the orbital and rotational angular frequencies, respectively. The amplitude and phase depends on the location of the source in the sky.

The Garching group computed the time difference using a FORTRAN code called EARTH,(export version 3.0 Max-Planck-Institut f*ü*r Radioastronomie, 1981). The results of the program were fitted to equation reported above giving the parameters: $a_O = 31.43$ s, $a_R = 5.00$ ms, $\Phi_O = 214$, and $\Phi_R = 313$. The time was referenced to the starting local time of the 100 h run.

The method to correct the Doppler shift used by the Garching group, which is alternative to interpolation of the input data, is to slowly vary the phase of the heterodyne frequency to follow the Doppler induced phase shift. The correction is made by simply multiplying the analytic signal with $exp(i\omega_o \delta t)$, where ω_o is the anticipated signal frequency near 2 or 4 kHz and δt is the time correction between the laboratory and barycentric frames. This can also be understood as a slow rotation of the complex plane so that the Doppler shifted signal remains stationary. The authors stress two important advantages of this procedure over interpolation:

- the phase shift can be calculated explicitly so that the Doppler shift correction can be made exact
- the correction can be performed on the reduced data. In the case of the Garching analysis ω_o is assumed to be known. However, in a blind frequency search it is possible to use the heterodyne frequency ω_m (instead of ω_o, the unknown signal frequency) to correct for the Doppler shift of all signals inside a limited bandwidth.

Data handling and final results

The complex heterodyne data compression was applied to the 100 h of data taken with the Garching interferometer. First, the original 10 kHz sampled data were complex heterodyned (sine and cosine) using two independent frequencies of 1968.658 and 3937.316 Hz. The channels were low-pass filtered and resampled at 4 Hz. Each channel, after data compression, consisted of about 1.5 million complex time samples over the 100 h. The 100 h data stream after data compression consisted of a complex time series sampled at 4 Hz. First they subdivided the data into many time slices made up of 2048 points which corresponds to a real time of 8.53 min. The number of resampled data points was chosen to be an integer power of 2 which facilitated use of the standard complex FFT subroutine. This time period was short enough so that the effect of Doppler shift and changing sensitivity to linearly polarized gravitational waves could be neglected. The time slices were chosen so that the noise level of the interferometer remained roughly constant over this time scale. They made the evaluation first with many 9 minute time slices. The length of the FFT was then increased up to 100 h. The shorter integration periods have the advantage that the corrections due to Doppler shift or changing antenna sensitivity are less important. In addition, the problems of inter-tape gaps and changing sensitivity could be avoided using time scales less than or equal to an hour. Shorter FFT's are also better when searching for unknown signals whose strength or frequency is time dependent. In general, the noise was found to decrease as the square root of the integration time for single FFT's and no unexpected difficulties were encountered with the introduction of gaps, Doppler shift correction, or demodulation for the two orthogonal polarizations. Concerning the analysis results the authors present only the outputs of the 9 minute and 100 h integration periods.

The 9 minutes spectra were "whitened" by multiplying with the inverse of the calculated filter attenuation of the last two Butterworth filters. Then a histogram was made for the normalized power spectrum of each independent 9 minute time slice. White noise, Gaussian in the time domain, should lead to a power spectrum with independent components distributed as a negative exponential. All power spectra were normalized so that the mean value was unity before accumulating the amplitude statistics.

The mean of all the power spectra corresponded to an average linear strain amplitude of $2.5 \cdot 10^{-20}$ for an observation time of 8.5 min. There was no apparent clustering near any particular frequency and there was no power greater than 14 times the mean value. In linear amplitude, this translates into a strain limit of about

$$h \simeq 9.4 \cdot 10^{-20}$$

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A further step is to combine the spectra in order to improve the SNR. However, the 100 h stream contained gaps and had a noise level which varied over time. Furthermore, one can no longer neglect the Doppler shift correction and the correction due to the changing sensitivity of the antenna to linearly polarized gravitational waves for long integration. A g. w. signal of arbitrary polarization can be decomposed into a superposition of the two orthogonal polarizations usually denoted $h_{\oplus}$ and $h_{\otimes}$. In most cases the best signal detection algorithm is to search independently for the two signals by amplitude demodulation using the sensitivity function of the antenna to the desired polarization. Thus, they made three different searches: search for a signal by making only the Doppler shift correction, dependent searches applying the amplitude modulation for $h_\oplus$ and $h_\otimes$. In all three cases, the power spectrum was "whitened". The whitened spectra were featureless and had amplitude statistics which followed very closely those predicted by the Rayleigh distribution. The mean strain amplitudes were

$$h \simeq 1.26 \cdot 10^{-21}$$

for the case where first no polarization was extracted, and

$$h_\oplus \simeq 2.18 \cdot 10^{-21} \qquad\qquad h_\otimes \simeq 2.22 \cdot 10^{-21}$$

for the two possible states of polarization. A normalized power level of 17 corresponds to a 95% confidence level that no false alarm due to the noise would occur in any one of the 1441792 bins of the spectrum.

No significant signals above the level of 17 were found. This threshold corresponds to limits which are larger by a factor of 4 on the strain amplitude than the mean values quoted above, or $9 \cdot 10^{-21}$ for either polarization.